Speed of Sound in the Mass Varying Neutrinos Scenario

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ABSTRACT

We discuss about the speed of sound squared in the Mass Varying Neutrinos scenario (MaVaNs). Recently, it was argued that the MaVaNs has a catastrophic instability which is the emergence of an imaginary speed of sound at the non-relativistic limit of neutrinos. As the result of this instability, the neutrino-acceleron fluid cannot act as the dark energy. However, it is found that the speed of sound squared in the neutrino-acceleron fluid could be positive in our model. We examine the speed of sound in two cases of the scalar potential. One is the small fractional power-law potential and another is the logarithmic one. The power-law potential model with the right-handed neutrinos gives a stable one.

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I INTRODUCTION

One of the most challenging questions in both cosmological and particle physics is the nature of the dark energy in the Universe. At the present epoch, the energy density of the Universe is dominated by a dark energy component, whose negative pressure causes the expansion of the Universe to accelerate. In order to clarify the origin of the dark energy, one has tried to understand the connection of the dark energy with particle physics.

In a scenario proposed by Fardon, Nelson and Winer (MaVaNs), relic neutrinos could form a negative pressure fluid and cause cosmic acceleration [1]. In this idea, an unknown scalar field which is called "acceleron" is introduced and neutrinos interact through a new scalar force. The acceleron field sits at the instantaneous minimum of its potential, and the cosmic expansion only modulates this minimum through changes in the neutrino density. Therefore the neutrino mass is given by the acceleron, in other words, it depends on its number density and changes with the evolution of the Universe. The cosmological parameter w and the dark energy also evolve with the neutrino mass. Those evolutions depend on a model of the scalar potential strongly. Typical examples of the potential have been discussed by Peccei [2].

The variable neutrino mass was considered at first in [3], and was discussed for neutrino clouds [4]. Ref. [5] considered coupling of the dark energy scalar, such as Quintessence to the neutrinos and discuss its impact on the neutrino mass limits from Baryogenesis. The MaVaNs scenario leads to interesting phenomenological results. The neutrino oscillations may be a probe of the dark energy [6, 7, 8]. The leptogenesis [9, 10], the cosmo MSW effect of neutrinos [11] and the solar neutrino [12, 13] have been studied in the context of this scenario. Cosmological discussions of the scenario are also presented [14, 15, 16, 17, 18, 19, 20]. The extension to the supersymmetry have been presented in ref. [21, 22]. This scenario is also discussed in the context of the texture of the neutrino mass matrix with three families [23].

Despite many implications of this scenario, ref. [24] showed that this scenario contains a catastrophic instability which occurs when neutrinos become non-relativistic. As neutrinos become non-relativistic, the speed of sound squared in the neutrino-acceleron fluid turns to be negative. As the result of this instability, neutrinos condense into neutrino nuggets, and thus cannot act as the dark energy. Here, it is important to say that some non-adiabatic

models [14, 15] do not suffer from this instability. However, we have found that the speed of sound squared in this fluid could be positive even though neutrinos are not enough relativistic and the neutrino-acceleron fluid is adiabatic. In order to realize the positive speed of sound squared, a constraint for the scalar potential is required.

The paper is organized as follows: in Sec.II, we summarize the MaVaNs scenario with three families. Sec.III presents discussions for the speed of sound in the hydrodynamic picture. Sec.IV presents a stable model in the MaVaNs scenario. Sec.V devotes to the summary.

II DARK ENERGY FROM MaVaNs

In the MaVaNs scenario, one considers a dark energy sector consisting an "acceleron" field, ϕ_a and a dark fermion, ψ_n. This sector couples to the standard model sector only through neutrinos. The dark energy is assumed to be the sum of the energy densities of neutrinos and a scalar potential for the acceleron:

$$\rho_{\rm DE} = \rho_\nu + V(\phi_a), \tag{1}$$

where the potential energy of the acceleron is responsible for the acceleration of the Universe and for the dynamical neutrino mass. The energy density for three generations of neutrinos and antineutrinos is generally given by

$$\rho_\nu = T^4 \sum_{i=1}^{3} F(\xi_i), \qquad \xi_i \equiv \frac{m_{\nu i}}{T}, \qquad F(\xi_i) \equiv \frac{1}{\pi^2}\int_0^\infty \frac{dy y^2\sqrt{y^2+\xi_i^2}}{e^y+1}, \tag{2}$$

where i denotes three families.[1]

In the scenario, $\rho_{\rm DE}$ is stationary with respect to variation in the neutrino mass. This stationary condition is represented by

$$\frac{\partial \rho_\nu}{\partial \sum_{i=1}^{3} m_{\nu i}} + \frac{\partial V(\phi_a(m_{\nu i}))}{\partial \sum_{i=1}^{3} m_{\nu i}} = 0. \tag{3}$$

[1]In ref. [1, 2], there is a more precise discussion about the Fermi factor in eq.(2). For our purpose it suffices to use this form.

If $\partial \sum m_{\nu i}/\partial \phi_a \neq 0$, this condition turns to

$$T^4 \sum_{i=1}^{3} \frac{\partial F}{\partial \xi_i} \frac{\partial \xi_i}{\partial \phi_a} + \frac{\partial V(\phi_a)}{\partial \phi_a} = 0. \tag{4}$$

Using the equation of energy conservation in the Robertson-Walker background and the above stationary condition, one can get the equation of state parameter w as follows:

$$w + 1 = \frac{[4 - h(T)]\,\rho_\nu}{3\rho_{\mathrm{DE}}}, \tag{5}$$

where

$$h(T) \equiv \frac{\displaystyle\sum_{i=1}^{3} \xi_i \frac{\partial F(\xi_i)}{\partial \xi_i}}{\displaystyle\sum_{j=1}^{3} F(\xi_j)}. \tag{6}$$

The speed of sound squared in the neutrino-acceleron fluid is given by

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}_{\mathrm{DE}}} = \frac{\dot{w}\rho_{\mathrm{DE}} + w\dot{\rho}_{\mathrm{DE}}}{\dot{\rho}_{\mathrm{DE}}} \tag{7}$$

where p is the pressure of the dark energy [24, 25]. Recently, it was argued that when neutrinos are non-relativistic, this speed of sound squared becomes negative in this scenario: $c_s^2 = (\partial \ln m_\nu / \partial \ln n_\nu) < 0$, where n_ν is the number density of neutrinos. The emergence of an imaginary speed of sound shows that the MaVaNs scenario with non-relativistic neutrinos is unstable, and thus the fluid in the scenario cannot act as the dark energy. However, it is found that the speed of sound squared in this fluid can be positive even though neutrinos are non-relativistic. Then, a constraint for the scalar potential is required.

III SPEED OF SOUND

At the non-relativistic limit of neutrinos, the energy density of neutrinos is given by

$$\rho_\nu = \sum_{i=1}^{3} m_{\nu i} n_\nu, \tag{8}$$

then the stationary condition eq.(3) is rewritten as follows:

$$n_\nu = -\frac{\partial V(\phi_a)}{\partial \displaystyle\sum_{i=1}^{3} m_{\nu i}}. \tag{9}$$

Now the dark energy density is given as

$$\rho_{\rm DE} = \sum_{i=1}^{3} m_{\nu i} n_\nu + V(\phi_a). \tag{10}$$

Using these relations, the speed of sound squared in the neutrino-acceleron fluid is negative: $c_s^2 = (\partial \ln m_\nu / \partial \ln n_\nu) < 0$ as shown in ref. [24]. In order to study c_s^2 quantitatively, we start with discussing the energy density of neutrinos in eq.(2). Taking account that ξ_i is much larger than 1 for non-relativistic neutrinos, the function $F(\xi_i)$ is expanded in terms of ξ_i^{-1} as:

$$F(\xi_i) \simeq \frac{\hat{n}_\nu}{T^3}\xi_i + a\frac{\hat{n}_\nu}{T^3}\frac{1}{\xi_i}, \tag{11}$$

where

$$\hat{n}_\nu \equiv \frac{T^3}{\pi^2}\int_0^\infty \frac{dyy^2}{e^y+1}, \qquad a \equiv \frac{\int_0^\infty \frac{dyy^4}{e^y+1}}{2\int_0^\infty \frac{dyy^2}{e^y+1}} \simeq 6.47. \tag{12}$$

Since the first term of the right hand side in eq.(10) is derived from the first term of the right hand side in eq.(11), the effect of the second term in eq.(11) should be added to the dark energy density as a correction. Due to this correction, it could be that the negative speed of sound squared turns to be positive under a condition, which is discussed in detail later.

The dark energy density including the correction term is given as follows:

$$\rho_{\rm DE} = \sum_{i=1}^{3} m_{\nu i}\hat{n}_\nu \left(1 + \frac{a}{\xi_i^2}\right) + V(\phi_a). \tag{13}$$

Then the stationary condition eq.(4) is described as

$$T\sum_{i=1}^{3}\left(\hat{n}_\nu - \frac{a\hat{n}_\nu}{\xi_i^2}\right)\frac{\partial \xi_i}{\partial \phi_a} + \frac{\partial V(\phi_a)}{\partial \phi_a} = 0. \tag{14}$$

The equation of state is generally given by eq.(5). At the non-relativistic limit, it is easy to see $h(T) = 1$. Because of the correction term, $h(T)$ is deviated from 1 as follows:

$$h(T) \equiv \frac{T^4 \sum_{i=1}^{3} \xi_i \dfrac{\partial F(\xi_i)}{\partial \xi_i}}{\sum_{j=1}^{3} F(\xi_j)} = \frac{1 - \dfrac{1}{\sum_{i=1}^{3}\xi_i}\left(\sum_{j=1}^{3}\dfrac{a}{\xi_j}\right)}{1 - \dfrac{1}{\sum_{k=1}^{3}\xi_k}\left(\sum_{l=1}^{3}\dfrac{a}{\xi_l}\right)} \simeq 1 - \frac{2}{\sum_{i=1}^{3}\xi_i}\left(\sum_{j=1}^{3}\frac{a}{\xi_j}\right). \tag{15}$$

Thus, the equation of state is given by

$$w + 1 = \frac{\hat{n}_\nu \sum_{i=1}^{3} \left(3m_{\nu i} + \frac{5aT}{\xi_i} \right)}{3\rho_{\rm DE}} \tag{16}$$

where we omitted the term of $\mathcal{O}(1/\xi^3)$. Using eqs.(13) and (16) and the stationary condition eq.(3), the speed of sound squared in the neutrino-acceleron fluid is described finally as follows (see Appendix):

$$\begin{aligned} c_s^2 &= \frac{\dot{w}\rho_{\rm DE} + w\dot{\rho}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \frac{\frac{\partial w}{\partial z}\rho_{\rm DE} + w\frac{\partial \rho_{\rm DE}}{\partial z}}{\frac{\partial \rho_{\rm DE}}{\partial z}} \\ &= \frac{\sum_{i=1}^{3} \frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu}{\sum_{i=1}^{3} m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}} + \frac{\frac{5}{3}a\hat{n}_\nu \sum_{i=1}^{3}\left(\frac{5T_0}{\xi_i} - \frac{T}{\xi_i^2}\frac{\partial m_{\nu i}}{\partial z} \right)}{\sum_{i=1}^{3} m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}}, \end{aligned} \tag{17}$$

where z is the redshift parameter, $z \equiv (T/T_0) - 1$ and "0" represents a value at the present epoch. We have taken the form differentiated by the redshift parameter instead of the time. The first term in the right hand side of eq.(17) is the leading term at the non-relativistic limit and is negative definite because of $\partial m_{\nu i}/\partial z < 0$ and $\partial \hat{n}_\nu/\partial z > 0$. The numerator of the second term, which is the correction term, is the positive definite. Thus, it is possible that the speed of sound squared is positive, in other words, the neutrino-acceleron fluid could be stable due to this term. From the eq.(17), it is easy to see that if the following relation is satisfied, the speed of sound squared becomes positive:

$$\sum_{i=1}^{3} \frac{\partial m_{\nu i}}{\partial z}\left(1 - \frac{5aT^2}{3m_{\nu i}^2} \right) + \frac{25aT_0^2(z+1)}{3}\sum_{i=1}^{3}\frac{1}{m_{\nu i}} > 0. \tag{18}$$

The sign of c_s^2 is determined by the magnitude of $\partial m_{\nu i}/\partial z$. Since the magnitude of $\partial m_{\nu i}/\partial z$ is model dependent, we will examine two cases of the scalar potential for the acceleron, one is the power-law potential and another is the logarithmic one in the next section.

IV MODELS

In the MaVaNs scenario, one needs a flat scalar potential [1]. Therefore, we will discuss the speed of sound in cases of the small fractional power-law potential and logarithmic one in

this section. The magnitude of $\partial m_{\nu i}/\partial z$ does not only depend on the scalar potential but also the coupling between neutrinos and the acceleron. Therefore, we consider two cases in eqs.(20) and (28) for this coupling.

A. The power-law potential

We take the small fractional power-law potential of the form

$$V(\phi_a) = A\left(\frac{\phi_a}{\phi_a^0}\right)^k, \quad k \ll 1, \tag{19}$$

where parameters A and k are fixed by the magnitude of the dark energy and the stationary condition at the present epoch, respectively.

Three left-handed neutrinos and a sterile neutrino

We take a Lagrangian of the form

$$\mathcal{L} = \bar{\nu}_{L\alpha} m_D^\alpha \psi_n + \lambda\phi_a\psi_n\psi_n + h.c.\ , \tag{20}$$

where ν_L and ψ_n are the left-handed and a sterile neutrino, respectively. Since we consider three families of active neutrinos, the mass matrix m_D is 3×1 matrix. Thus the neutrino mass matrix is given by

$$M_\nu = \frac{m_D m_D^T}{\lambda\phi_a}, \tag{21}$$

where we assume $m_D^\alpha \ll \lambda\phi_a$. After diagonarizing this matrix, we can find mass eigenvalues of neutrinos as follows:

$$m_{\nu i} = \frac{M_i^2}{\lambda\phi_a}, \tag{22}$$

which gives

$$\xi_i = \frac{M_i^2}{\lambda\phi_a T}. \tag{23}$$

Using eqs.(1), (19) and (23), we have the redshift dependence of the acceleron from the stationary condition eq.(4) as

$$\phi_a \simeq \frac{-Ak + G(z)}{2\lambda a\hat{n}_\nu T^2 \sum_{l=1}^{3}(1/M_l^2)}, \quad G(z) \equiv \sqrt{A^2k^2 + 4a\hat{n}_\nu^2 T^2 \sum_{j=1}^{3}(1/M_j^2)\sum_{k=1}^{3} M_k^2}, \tag{24}$$

where the leading term is taken after expanding $\partial V/\partial\phi_a$ by k. Thus, the redshift dependence of neutrino masses is given by

$$\sum_{i=1}^{3}\frac{\partial m_{\nu i}}{\partial z} = -\sum_{i=1}^{3}\frac{M_i^2}{\lambda\phi_a^2}\frac{\partial\phi_a}{\partial z}$$
$$= -\frac{4a^2\hat{n}_\nu^2T^4\left[\sum_{l=1}^{3}(1/M_l^2)\right]^2\sum_{i=1}^{3}M_i^2}{(z+1)[-Ak+G(z)]^2}\left[\frac{8\hat{n}_\nu\sum_{l=1}^{3}M_l^2}{G(z)} - \frac{5[-Ak+G(z)]}{2a\hat{n}_\nu T^2\sum_{l=1}^{3}(1/M_l^2)}\right], \quad (25)$$

where the right hand side is negative and its absolute value is the increasing function of k.

At the present epoch, values of some parameters are given by

$$T_0 \simeq 1.69\times 10^{-4}(\mathrm{eV}), \quad A \simeq 2.99\times 10^{-11}(\mathrm{eV}^4), \quad \hat{n}_\nu^0 \simeq 8.82\times 10^{-13}(\mathrm{eV}^3), \quad (26)$$

and we take the following typical masses at the present epoch:

$$m_{\nu 1}^0 = 0.0045(\mathrm{eV}), \quad m_{\nu 2}^0 = 0.01(\mathrm{eV}), \quad m_{\nu 3}^0 = 0.05(\mathrm{eV}), \quad (27)$$

which lead to $\Delta m^2_{\mathrm{atm}} = 2.4\times 10^{-3}(\mathrm{eV}^2)$ and $\Delta m^2_{\mathrm{sun}} = 8.0\times 10^{-5}(\mathrm{eV}^2)$. We numerically evaluate the relation (18) by putting values of (26) and (27). We find that k has to be smaller than 5.5×10^{-5} to satisfy the relation (18). However this value of k is unfavored in the phenomenology of the neutrino experiments. The value of k is related with neutrino masses through the stationary condition eq.(14). Actually, the k which is smaller than 5.5×10^{-5} leads to $\sum m_{\nu i} \sim \mathcal{O}(10^{-4})(\mathrm{eV})$. In the case of $10^{-4} < k < 10^{-2}$, $\sum m_{\nu i} \sim \mathcal{O}(10^{-3} \sim 10^{-1})(\mathrm{eV})$ is expected. Therefore, this model including the relation (22) and the scalar potential (19) is unfavored.

Three left- and right-handed neutrinos and a sterile neutrino

We add usual right-handed neutrinos to the Lagrangian (20)

$$\mathcal{L} = \bar{\nu}_{L\alpha} m_D^\alpha \psi_n + \lambda\phi_a\psi_n\psi_n + \bar{\nu}_{L\alpha} M_D^{\alpha\beta}\nu_{R\beta} + \nu_{R\alpha}^T M_R^{\alpha\beta} C^{-1}\nu_{R\beta} + h.c.\ , \quad (28)$$

where ν_R is the right-handed neutrinos and both M_D and M_R are 3×3 matrix. Then, the neutrino mass matrix is given as the 7×7 matrix

$$M = \begin{pmatrix} 0 & m_D & M_D \\ m_D^T & \lambda\phi_a & 0 \\ M_D^T & 0 & M_R \end{pmatrix}, \quad (29)$$

in the (ν_L, ψ_n, ν_R) basis. We take the right-handed Majorana mass scale to be much higher than the Dirac neutrino mass scale, and assume $m_D^\alpha \ll \lambda\phi_a$. Then, the effective neutrino mass matrix is approximately given by

$$M_\nu = M_D M_R^{-1} M_D^T + \frac{m_D m_D^T}{\lambda\phi_a}. \tag{30}$$

The first term in the right hand side of eq.(30) is the time-independent neutrino seesaw mass matrix, which is denoted by $\tilde{M}_\nu$, and so it depends on the flavor model of neutrinos. In this case, since the neutrino mass matrix reduces to eq.(21) at the limit of $\tilde{M}_\nu \to 0$, we assume that $\tilde{M}_\nu$ dominates the effective neutrino mass matrix M_ν. Then we can describe generally mass eigenvalues in the first order perturbation as follows [23]:

$$m_{\nu i} = \tilde{m}_{\nu i} + c_i \frac{M_i^2}{\lambda\phi_a}, \tag{31}$$

where c_i is a coefficient of order 1 depending on the model of families.[2] Using this relation and eq.(19), we have the redshift dependence of the acceleron from the stationary condition eq.(14) as

$$\phi_a \simeq \frac{\hat{n}_\nu \sum_{i=1}^{3} c_i M_i^2 \left(1 - \dfrac{aT^2}{\tilde{m}_{\nu i}^2}\right)}{\lambda k A}, \tag{32}$$

where the first term of the left hand side in eq.(14) was expanded by the second term of the right hand side in eq.(31). Thus, the redshift dependence of neutrino masses is given by

$$\sum_{i=1}^{3} \frac{\partial m_{\nu i}}{\partial z} = -\frac{kA \sum_{i=1}^{3} c_i M_i^2 \sum_{j=1}^{3} M_j^2 \left(3 - \dfrac{5aT^2}{\tilde{m}_{\nu j}^2}\right)}{\hat{n}_\nu (z+1) \left[\sum_{k=1}^{3} M_k^2 \left(1 - \dfrac{aT^2}{\tilde{m}_{\nu k}^2}\right)\right]^2}. \tag{33}$$

The magnitude of the first term of the left hand side in eq.(18) is nearly equal to $kA/\hat{n}_\nu$ at the present epoch. In order to realize the positive speed of sound squared, k has to be smaller than 5.19×10^{-6}. When we assume that the magnitude of the second term of the right hand side in eq.(31) is 0.1 percent of the first term, we obtain $k = 1.90 \times 10^{-6}$ which reproduce observed values of neutrino masses. Therefore, the model including the scalar potential (19) and the relation between neutrino masses and the acceleron (31) is favored in the MaVaNs scenario and can act as the dark energy.

[2] The values of c_i are given in a specific flavor symmetry as in ref. [23].

B. The logarithmic potential

We take the logarithmic scalar potential of the form

$$V(\phi_a) = B \ln\left(\frac{\phi_a}{\mu}\right), \tag{34}$$

$$B \simeq 5.68 \times 10^{-14} (\mathrm{eV}^4), \tag{35}$$

where we use values of (26) and (27) to fix the value of B.

We will consider two cases of the coupling between neutrinos and the acceleron as well as the case of the small fractional potential.

Three left-handed neutrinos and a sterile neutrino

We take Lagrangian in eq.(20), and thus, neutrino masses in eq.(22). The stationary condition of eq.(14) gives the redshift dependence of the acceleron:

$$\phi_a \simeq \frac{-B + H(z)}{2\lambda a \hat{n}_\nu T^2 \sum_{l=1}^{3} (1/M_l^2)}, \qquad H(z) \equiv \sqrt{B^2 + 4a\hat{n}_\nu^2 T^2 \sum_{j=1}^{3}(1/M_j^2) \sum_{k=1}^{3} M_k^2}, \tag{36}$$

and thus the redshift dependence of neutrino masses is given by

$$\sum_{i=1}^{3} \frac{\partial m_{\nu i}}{\partial z} = -\frac{4a^2 \hat{n}_\nu^2 T^4 \left[\sum_{l=1}^{3}(1/M_l^2)\right]^2 \sum_{i=1}^{3} M_i^2}{(z+1)[-B + H(z)]^2} \left[\frac{8\hat{n}_\nu \sum_{l=1}^{3} M_l^2}{H(z)} - \frac{5\{-B + H(z)\}}{2a\hat{n}_\nu T^2 \sum_{l=1}^{3}(1/M_l^2)}\right]. \tag{37}$$

Using values of eqs.(26), (27) and (35), the first term of the left hand side in eq.(18) is $-\mathcal{O}(10^{-1})$ (eV), however the second term is $\mathcal{O}(10^{-4})$(eV) at the present epoch. Thus the speed of sound squared becomes negative, in other words, one cannot build a stable MaVaNs model including the form of the neutrino mass like as eq.(22).

Three left- and right-handed neutrinos and a sterile neutrino

We take Lagrangian in eq.(28), and thus, neutrino masses in eq.(31). The redshift dependence of the acceleron is given as:

$$\phi_a \simeq \frac{\hat{n}_\nu \sum_{i=1}^{3} c_i M_i^2 \left(1 - \frac{aT^2}{\tilde{m}_{\nu i}^2}\right)}{\lambda B}, \tag{38}$$

and thus, the redshift dependence of neutrino masses is given by

$$\sum_{i=1}^{3}\frac{\partial m_{\nu i}}{\partial z}=-\frac{B\sum_{i=1}^{3}c_iM_i^2\sum_{j=1}^{3}M_j^2\left(3-\frac{5aT^2}{\tilde{m}_{\nu j}^2}\right)}{\hat{n}_\nu(z+1)\left[\sum_{k=1}^{3}M_k^2\left(1-\frac{aT^2}{\tilde{m}_{\nu k}^2}\right)\right]^2}. \tag{39}$$

Using values of eqs.(26) and (27), we can estimate the redshift dependence of neutrino masses. Since $\tilde{m}_{\nu i}$ dominate the neutrino mass, we assume $\tilde{m}_{\nu i}\sim m_{\nu i}$. In this case, the first term of the left hand side in eq.(18) is $-\mathcal{O}(10^{-3})$(eV), however the second term is $\mathcal{O}(10^{-4})$(eV). Thus, the speed of sound squared becomes negative as well as the previous case. We conclude that the speed of sound squared in the neutrino-acceleron fluid becomes negative when neutrinos are non-relativistic for the logarithmic scalar potential in eq.(34) and neutrino masses like as eqs.(22) and (31).

V SUMMARY

We have discussed about the speed of sound squared in the neutrino-acceleron fluid, and tried to find a stable MaVaNs model in which the fluid is adiabatic. In order to examine c_s^2 quantitatively, we have taken two types of the scalar potential for the acceleron. One is the small fractional power-law potential and another is the logarithmic one. Furthermore, we have studied about two types of models which have different couplings between neutrinos and the accerelon.

In our analysis, models including the logarithmic scalar potential are unstable and cannot act as the dark energy because the speed of sound squared in the neutrino-acceleron fluid becomes negative. However, models with the small fractional power-law potential can avoid this instability. The model including only three left-handed neutrinos and a sterile neutrino avoids this instability but does not reproduce the observed neutrino masses. On the other hand, the model including the right-handed neutrinos reproduces the observed neutrino masses and realizes the positive speed of sound squared. Neutrino masses in this model have the time-independent component from the seesaw mechanism, which was assumed to be dominant in the effective neutrino mass. Therefore, it is easy to reconcile these neutrino masses with observed ones. Due to this time-independent mass, this model becomes viable.

Appendix

The derivation of the speed of sound squared in eq.(17) is presented in this Appendix.

The energy density for three generations of neutrinos and antineutrinos is generally given by

$$\rho_\nu = T^4 \sum_{i=1}^{3} F(\xi_i), \quad \xi_i \equiv \frac{m_{\nu i}}{T}, \quad F(\xi_i) \equiv \frac{1}{\pi^2}\int_0^\infty \frac{dy y^2 \sqrt{y^2+\xi_i^2}}{e^y+1}. \tag{40}$$

As neutrinos become non-relativistic, ξ_i is much larger than 1. Therefore, the function $F(\xi_i)$ is expanded in terms of ξ_i^{-1} as:

$$\begin{aligned} F(\xi_i) &= \frac{1}{\pi^2}\int_0^\infty \frac{dy y^2}{e^y+1}\xi_i\sqrt{\left(\frac{y}{\xi i}\right)^2+1} \\ &\simeq \frac{1}{\pi^2}\int_0^\infty \frac{dy y^2}{e^y+1}\xi_i\left[\frac{1}{2}\left(\frac{y}{\xi_i}\right)^2+1\right] \\ &= \frac{\xi_i}{\pi^2}\int_0^\infty \frac{dy y^2}{e^y+1} + \frac{1}{2\pi^2\xi_i}\int_0^\infty \frac{dy y^4}{e^y+1} \\ &= \frac{\hat{n}_\nu}{T^3}\xi_i + \frac{1}{2\pi^2\xi_i}\int_0^\infty \frac{dy y^4}{e^y+1} \\ &= \frac{\hat{n}_\nu}{T^3}\xi_i + a\frac{\hat{n}_\nu}{T^3}\frac{1}{\xi_i}, \end{aligned} \tag{41}$$

where

$$\hat{n}_\nu \equiv \frac{T^3}{\pi^2}\int_0^\infty \frac{dy y^2}{e^y+1}, \quad a \equiv \frac{\int_0^\infty \frac{dy y^4}{e^y+1}}{2\int_0^\infty \frac{dy y^2}{e^y+1}} \simeq 6.47, \tag{42}$$

and thus, we get

$$\rho_\nu = \sum_{i=1}^{3} m_{\nu i}\hat{n}_\nu + \sum_{i=1}^{3} a\hat{n}_\nu \frac{T}{\xi_i} = \sum_{i=1}^{3} m_{\nu i}\hat{n}_\nu\left(1+\frac{a}{\xi_i^2}\right), \tag{43}$$

$$\rho_{\rm DE} = \rho_\nu + V(\phi_a) = \sum_{i=1}^{3} m_{\nu i}\hat{n}_\nu\left(1+\frac{a}{\xi_i^2}\right) + V(\phi_a), \tag{44}$$

$$\frac{\partial \rho_{\rm DE}}{\partial z} = \sum_{i=1}^{3}\left[\left(\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu + m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}\right)\left(1+\frac{a}{\xi_i^2}\right) - m_{\nu i}\hat{n}_\nu\frac{2a}{\xi_i^3}\frac{\partial \xi_i}{\partial z}\right] + \frac{\partial V(\phi_a)}{\partial z}. \tag{45}$$

The stationary condition eq.(3) leads to the relation:

$$\frac{\partial V(\phi_a)}{\partial z}\frac{\partial z}{\partial \sum_{i=1}^{3} m_{\nu i}} = -\frac{\partial}{\partial \sum_{i=1}^{3} m_{\nu i}}\left[\sum_{j=1}^{3} m_{\nu j}\hat{n}_\nu\left(1+\frac{a}{\xi_j^2}\right)\right]$$

$$
\begin{aligned}
&= -\hat{n}_\nu - \frac{\partial z}{\partial \sum_{i=1}^{3} m_{\nu i}} \frac{\partial}{\partial z}\left(\sum_{j=1}^{3} m_{\nu j}\hat{n}_\nu \frac{a}{\xi_j^2}\right) \\
&= -\hat{n}_\nu - \frac{\partial z}{\partial \sum_{i=1}^{3} m_{\nu i}} \left[\sum_{j=1}^{3}\left(\frac{\partial m_{\nu j}}{\partial z}\hat{n}_\nu \frac{a}{\xi_j^2} + m_{\nu j}\frac{\partial \hat{n}_\nu}{\partial z}\frac{a}{\xi_j^2} - 2m_{\nu j}\hat{n}_\nu \frac{a}{\xi_j^3}\frac{\partial \xi_j}{\partial z}\right)\right],
\end{aligned}
\tag{46}
$$

and thus, we have

$$
\frac{\partial V(\phi_a)}{\partial z} = \sum_{i=1}^{3}\left[\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu - a\left(\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu \frac{1}{\xi_i^2} + m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}\frac{1}{\xi_i^2} - 2m_{\nu i}\hat{n}_\nu \frac{1}{\xi_i^3}\frac{\partial \xi_i}{\partial z}\right)\right]. \tag{47}
$$

Using eqs.(45) and (47), the redshift dependence of the dark energy is

$$
\begin{aligned}
\frac{\partial \rho_{\mathrm{DE}}}{\partial z} &= \sum_{i=1}^{3}\left(\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu + m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}\right)\left(1+\frac{a}{\xi_i^2}\right) - \sum_{i=1}^{3} m_{\nu i}\hat{n}_\nu \frac{2a}{\xi_i^3}\frac{\partial \xi_i}{\partial z} \\
&\quad + \sum_{i=1}^{3}\left[\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu - a\left(\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu \frac{1}{\xi_i^2} + m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}\frac{1}{\xi_i^2} - 2m_{\nu i}\hat{n}_\nu \frac{1}{\xi_i^3}\frac{\partial \xi_i}{\partial z}\right)\right] \\
&= \sum_{i=1}^{3} m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}.
\end{aligned}
\tag{48}
$$

The equation of state parameter w is

$$
w+1 = \frac{[4-h(T)]\rho_\nu}{3\rho_{\mathrm{DE}}}, \tag{49}
$$

where

$$
h(T) \equiv \frac{\sum_{i=1}^{3} \xi_i \frac{\partial F(\xi_i)}{\partial \xi_i}}{\sum_{j=1}^{3} F(\xi_j)}. \tag{50}
$$

Using eq.(41),

$$
\begin{aligned}
h(T) &= \frac{\sum_{i=1}^{3} \xi_i\left(\frac{\hat{n}_\nu}{T^3} - a\frac{\hat{n}_\nu}{T^3}\frac{1}{\xi_i^2}\right)}{\sum_{j=1}^{3}\left(\frac{\hat{n}_\nu}{T^3}\xi_j + a\frac{\hat{n}_\nu}{T^3}\frac{1}{\xi_j}\right)} \\
&= \frac{\sum_{i=1}^{3}\xi_i - \sum_{j=1}^{3}\frac{a}{\xi_j}}{\sum_{k=1}^{3}\xi_k + \sum_{l=1}^{3}\frac{a}{\xi_l}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1-\dfrac{1}{\sum_{i=1}^{3}\xi_i}\left(\sum_{j=1}^{3}\dfrac{a}{\xi_j}\right)}{1-\dfrac{1}{\sum_{k=1}^{3}\xi_k}\left(\sum_{l=1}^{3}\dfrac{a}{\xi_l}\right)} \\
&\simeq 1-\frac{2}{\sum_{i=1}^{3}\xi_i}\left(\sum_{j=1}^{3}\frac{a}{\xi_j}\right). \qquad (51)
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
w+1 &= \frac{\left[3+\dfrac{2}{\sum_{i=1}^{3}\xi_i}\left(\sum_{j=1}^{3}\dfrac{a}{\xi_j}\right)\right]\left(\sum_{k=1}^{3}m_{\nu k}\hat{n}_\nu+\sum_{l=1}^{3}a\hat{n}_\nu\dfrac{T}{\xi_l}\right)}{3\rho_{\mathrm{DE}}} \\
&= \frac{3\sum_{i=1}^{3}m_{\nu i}\hat{n}_\nu+3\sum_{i=1}^{3}a\hat{n}_\nu\dfrac{T}{\xi_i}+\dfrac{2}{\sum_{i=1}^{3}\xi_i}\left(\sum_{j=1}^{3}\dfrac{a}{\xi_j}\right)\sum_{k=1}^{3}m_{\nu k}\hat{n}_\nu}{3\rho_{\mathrm{DE}}} \\
&\quad +\frac{\dfrac{2}{\sum_{i=1}^{3}\xi_i}\left(\sum_{j=1}^{3}\dfrac{a}{\xi_j}\right)\sum_{k=1}^{3}a\hat{n}_\nu\dfrac{T}{\xi_k}}{3\rho_{\mathrm{DE}}}, \qquad (52)
\end{aligned}
$$

where the last term is negligible small because ξ_i is much larger than 1. Thus we get

$$
\begin{aligned}
\rho_{\mathrm{DE}}(w+1) &= \frac{3\sum_{i=1}^{3}m_{\nu i}\hat{n}_\nu+3\sum_{i=1}^{3}a\hat{n}_\nu\dfrac{T}{\xi_i}+\dfrac{2T}{\sum_{i=1}^{3}m_{\nu i}}\left(\sum_{j=1}^{3}\dfrac{a}{\xi_j}\right)\sum_{k=1}^{3}m_{\nu k}\hat{n}_\nu}{3} \\
&= \sum_{i=1}^{3}\left(m_{\nu i}\hat{n}_\nu+\frac{5a\hat{n}_\nu T}{3\xi_i}\right). \qquad (53)
\end{aligned}
$$

Differentiating eq.(53) by the redshift parameter, we have

$$
\frac{\partial\rho_{\mathrm{DE}}}{\partial z}w+\frac{\partial\rho_{\mathrm{DE}}}{\partial z}+\rho_{\mathrm{DE}}\frac{\partial w}{\partial z}=\sum_{i=1}^{3}\left[\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu+m_{\nu i}\frac{\partial\hat{n}_\nu}{\partial z}+\frac{5a}{3}\left(\frac{\partial\hat{n}_\nu}{\partial z}\frac{T}{\xi_i}+\frac{\partial T}{\partial z}\frac{\hat{n}_\nu}{\xi_i}-\frac{\hat{n}_\nu T}{\xi_i^2}\frac{\partial\xi_i}{\partial z}\right)\right], \qquad (54)
$$

which leads to

$$\begin{aligned}\frac{\partial \rho_{\rm DE}}{\partial z}w+\rho_{\rm DE}\frac{\partial w}{\partial z} &= \sum_{i=1}^{3}\left[\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu+m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}+\frac{5a}{3}\left(\frac{3\hat{n}_\nu}{z+1}\frac{T}{\xi_i}+T_0\frac{\hat{n}_\nu}{\xi_i}-\frac{\hat{n}_\nu T}{\xi_i^2}\frac{\partial \xi_i}{\partial z}\right)\right]-\frac{\partial \rho_{\rm DE}}{\partial z}\\ &= \sum_{i=1}^{3}\left[\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu+m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}+\frac{5a\hat{n}_\nu}{3}\left(\frac{4T_0}{\xi_i}-\frac{T}{\xi_i^2}\frac{\partial \xi_i}{\partial z}\right)\right]-\frac{\partial \rho_{\rm DE}}{\partial z}. \qquad (55)\end{aligned}$$

Then, using the relation (48), we have

$$\frac{\partial \rho_{\rm DE}}{\partial z}w+\rho_{\rm DE}\frac{\partial w}{\partial z}=\sum_{i=1}^{3}\left[\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu+m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}+\frac{5a\hat{n}_\nu}{3}\left(\frac{4T_0}{\xi_i}-\frac{T}{\xi_i^2}\frac{\partial \xi_i}{\partial z}\right)\right]-\sum_{i=1}^{3}m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}. \qquad (56)$$

Since the speed of sound squared in the dark energy is given by

$$\begin{aligned}c_s^2 &\equiv \frac{\partial p}{\partial \rho_{\rm DE}}\\ &= \frac{\partial (w\rho_{\rm DE})}{\partial \rho_{\rm DE}}\\ &= \frac{\frac{\partial \rho_{\rm DE}}{\partial z}w+\rho_{\rm DE}\frac{\partial w}{\partial z}}{\frac{\partial \rho_{\rm DE}}{\partial z}}, \qquad (57)\end{aligned}$$

using eqs.(48) and (56), we get finally

$$\begin{aligned}c_s^2 &= \frac{\sum_{i=1}^{3}\left[\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu+m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}+\frac{5a\hat{n}_\nu}{3}\left(\frac{4T_0}{\xi_i}-\frac{T}{\xi_i^2}\frac{\partial \xi_i}{\partial z}\right)\right]-\sum_{i=1}^{3}m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}}{\sum_{i=1}^{3}m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}}\\ &= \frac{\sum_{i=1}^{3}\frac{\partial m_{\nu i}}{\partial z}\hat{n}_\nu}{\sum_{i=1}^{3}m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}}+\frac{\frac{5}{3}a\hat{n}_\nu\sum_{i=1}^{3}\left(\frac{5T_0}{\xi_i}-\frac{T}{\xi_i^2}\frac{\partial m_{\nu i}}{\partial z}\right)}{\sum_{i=1}^{3}m_{\nu i}\frac{\partial \hat{n}_\nu}{\partial z}}. \qquad (58)\end{aligned}$$

It is easy to see that if the following relation is satisfied, the speed of sound squared becomes positive:

$$\sum_{i=1}^{3}\frac{\partial m_{\nu i}}{\partial z}\left(1-\frac{5aT^2}{3m_{\nu i}^2}\right)+\frac{25aT_0^2(z+1)}{3}\sum_{i=1}^{3}\frac{1}{m_{\nu i}}>0. \qquad (59)$$

References

[1] R. Fardon, A. E. Nelson and N. Weiner, J. Cosmol. Astropart. Phys. **10**, 005 (2004).

[2] R. D. Peccei, Phys. Rev. D **71**, 023527 (2005).

[3] M. Kawasaki, H. Murayama and T. Yanagida, Mod. Phys. Lett. A **7**, 563 (1992).

[4] G. J. Stephenson, T. Goldman and B. H. J. McKellar, Int. J. Mod. Phys. A **13**, 2765 (1998); Mod. Phys. Lett. A **12**, 2391 (1997).

[5] P. Gu, X-L. Wang and X-Min. Zhang, Phys. Rev. D **68**, 087301 (2003).

[6] D. B. Kaplan, A. E. Nelson, N. Weiner, Phy. Rev. Lett. **93**, 091801 (2004).

[7] V. Barger, D. Marfatia and K. Whisnant, hep-ph/0509163.

[8] P-H. Gu, X-J. Bi, B. Feng, B-L. Young and X. Zhang, hep-ph/0512076.

[9] X-J. Bi, P. Gu, X-L. Wang and X-Min. Zhang, Phys. Rev. D **69**, 113007 (2004).

[10] P. Gu and X-J. Bi, Phys. Rev. D **70**, 063511 (2004).

[11] P. Q. Hung and H. Päs, Mod. Phys. Lett. A **20**, 1209 (2005).

[12] V. Barger, P. Huber and D. Marfatia, Phys. Rev. Lett. **95**, 211802 (2005).

[13] M. Cirelli and M. C. Gonzalez-Garcia and C. Peña-Garay, Nucl. Phys. B **719**, 219 (2005).

[14] X-J. Bi, B. Feng, H. Li and X-Min. Zhang, hep-ph/0412002.

[15] A. W. Brookfield, C. van de Bruck, D. F. Mota and D. Tocchini-Valentini, astro-ph/0503349.

[16] R. Horvat, astro-ph/0505507;
R. Barbieri, L. J. Hall, S. J. Oliver and A. Strumia, Phys. Lett. B **625**, 189 (2005).

[17] N. Weiner and K. Zurek, hep-ph/0509201.

[18] H. Li, B. Feng, J-Q. Xia and X-Min. Zhang, astro-ph/0509272.

[19] A. W. Brookfield, C. van de Bruck, D. F. Mota and D. Tocchini-Valentini, astro-ph/0512367.

[20] P-H. Gu, X-J. Bi and X. Zhang, hep-ph/0511027.

[21] R. Takahashi and M. Tanimoto, hep-ph/0507142.

[22] R. Fardon, A. E. Nelson and N. Weiner, hep-ph/0507235.

[23] M. Honda, R. Takahashi and M. Tanimoto, hep-ph/0510018.

[24] N. Afshordi, M. Zaldarriaga and K. Kohri, Phys. Rev. D **72**, 065024 (2005).

[25] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. **215**, 203 (1992).